

Palatini wormholes and energy conditions from the prism of General Relativity

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Wormholes are hypothetical shortcuts in spacetime that in General Relativity unavoidably violate all of the pointwise energy conditions. In this paper, we consider several wormhole spacetimes that, as opposed to the standard *designer* procedure frequently employed in the literature, arise directly from gravitational actions including additional terms resulting from contractions of the Ricci tensor with the metric, and which are formulated assuming independence between metric and connection (Palatini approach). We reinterpret such wormhole solutions under the prism of General Relativity and study the matter sources that thread them. We discuss the size of violation of the energy conditions in different cases, and how this is related to the same spacetimes when viewed from the modified gravity side.

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I. INTRODUCTION

There exist numerous exact solutions of the Einstein field equations in which the physical relevance is not always clear and, consequently, are often regarded as exotic [1]. For instance, a well known example is the static and spherically symmetric Schwarzschild solution, where the importance of this geometry was not fully understood until the observational discovery of very compact objects such as neutron stars. In the 1960s, the Schwarzschild solution began to be considered seriously not only as an approximate description of the exterior geometry of (non-rotating) stars but also as the final outcome of gravitational collapse [2]. Nowadays, black holes are a hot topic of research, such as the sources of gravitational waves [3], and are no longer regarded as exotic solutions.

Wormholes are also regarded as an exotic family of solutions of Einstein's equations [4–6], and have been extensively analyzed for a number of reasons. From a quantum gravitational perspective, they are seen as a natural consequence if the topology of spacetime fluctuates in time [7]. Recently, a new connection between specific types of (non-traversable) wormholes [8] and quantum systems was introduced with relevant implications in high energy physics [9]. From an astrophysical and cosmological viewpoint, the late-time cosmic speedup [10] suggests that the source driving the expansion [11] could be compatible with the kind of matter necessary to generate traversable wormholes. In fact, several cosmological

models with exotic sources supporting wormholes have been proposed, including phantom energy [12], Chaplygin gas [13] or its generalizations [14]. This has led many researchers to explore the astrophysical consequences and characterization of macroscopic wormholes [15].

In the context of General Relativity (GR) wormhole spacetimes are specifically designed, where Einstein's equations are solved in a reversed manner. This means that a suitable well behaved geometry is defined *a priori* and the matter sources that generate it are derived *a posteriori*. The outstanding feature was the requirement of *exotic matter*, defined as matter that violates the null energy condition, i.e., $T_{\mu\nu}k^\mu k^\nu < 0$, where $T_{\mu\nu}$ is the stress-energy tensor and k^μ is any null vector [4, 5, 16]. Although exotic matter is considered classically unviable, there are experimental evidences of such energy conditions violations in several systems where quantum effects such as the Casimir effect, Hawking evaporation or vacuum polarization take place [5, 17]. Therefore, since the existence of exotic matter is classically a problematic issue, it is particularly important to find solutions which minimize the violation of the energy conditions. In fact, the amount of violation can be made infinitesimally small by choosing the geometry of the wormhole in a very specific and appropriate way [18]. The thin-shell formalism is another approach to minimize the exotic matter, as the latter is now localized on the thin-shell [19, 20].

However, in the context of modified gravity, it was shown explicitly that the matter threading the wormhole throat may, in principle, satisfy all of the energy conditions, and it is the higher order curvature terms, interpreted as a gravitational fluid, that support the wormhole geometry [21]. In fact, a plethora of wormhole solutions beyond GR have been found in a variety of theories, such as in conformal Weyl gravity [22], Kaluza-

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Klein [23], Brans-Dicke [24], Gauss-Bonnet [25], Lovelock [26], braneworlds [27], Hořava-Lifshitz [28], Eddington-inspired Born-Infeld [29], $f(R)$ gravity [30] and extensions [31], modified teleparallelism framework [32] or Einstein-Cartan theory [33], amongst others.

In this context, extensions of GR formulated in the metric-affine (or Palatini) approach are of particular interest. In theories of the $f(R)$ type, for instance, wormhole solutions turn out to be quite common and may arise without the need for exotic matter [34, 35], as mentioned above. This also occurs in other Palatini extensions, such as quadratic gravity, $R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}$, or the Born-Infeld gravity theory, which is defined through an unconventional square root form [36]. These wormholes can be supported by standard matter sources such as electric fields, which satisfy all the energy conditions. The case of electric fields is of particular interest as in GR such sources are unable to sustain wormholes [37]. Moreover, a self-gravitating electric field with a wormhole structure is an explicit realization of the notion of the geon introduced by Wheeler [7].

The purpose of this work is to interpret some of the wormhole solutions obtained in gravitational theories formulated in the Palatini approach from the prism of standard GR. This means that considering specific wormhole solutions, and taking into account the modified Einstein equation, it is then possible to interpret its associated effective stress-energy tensor from a standard perspective and determine if it violates the generalized energy conditions [21]. This approach could shed new light on the kind of energy sources that in the context of GR could generate these types of solutions. More specifically, this strategy is naturally motivated in the context of wormholes in Palatini $f(R)$ theories (see Sec. II A below for details). This is so because the field equations of such theories can be written in an Einstein-like form, $G_{\mu\nu}(g) = \kappa^2 \tau_{\mu\nu}$, with $\kappa^2 = 8\pi G_N$ and $\tau_{\mu\nu}$ represents an effective stress-energy tensor whose form depends on the $f(R)$ Lagrangian and the particular matter source considered. For linear $f(R)$, the case of GR, one finds $\tau_{\mu\nu} = T_{\mu\nu}$ but for a nonlinear $f(R)$, $\tau_{\mu\nu}$ picks up new terms that depend on the function $f(R)$ and its derivatives, being R a (typically nonlinear) function of the trace of $T_{\mu\nu}$. As a result, in vacuum, $\tau_{\mu\nu}$ boils down to zero or to a cosmological constant term, depending on the particular $f(R)$ Lagrangian chosen.

The existence of new contributions on the right-hand side of the equations generated by the matter fields via the function $f(R)$ justifies that standard matter sources, such as (nonlinear) electric fields, can generate wormholes. The modified field equations of Palatini theories, therefore, can somehow be seen as inducing modifications in the matter sector that allow standard matter fields to violate the energy conditions. We are thus interested in understanding how and where these violations occur. The reinterpretation of these wormhole solutions from the prism of GR might be useful to understand their properties from a more standard perspective.

The paper is organized as follows. We begin by introducing the basics of Palatini theories in Sec. II. In Sec. III we review the standard wisdom on wormholes in the context of GR. In Sec. IV we present Palatini wormholes, namely, wormholes obtained in Born-Infeld gravity formulated in the Palatini approach, and discuss them under the prism of GR, identifying the matter source threading the wormhole and discussing the violation of the energy conditions. In Sec. V this analysis is extended to wormholes obtained in the context of $f(R)$ gravities, and we conclude in Sec. VI with a summary and discussion.

II. PALATINI THEORIES OF GRAVITY

The wormhole solutions we are going to study have been found in theories of gravity formulated à la Palatini, i.e., assuming that the metric and the connection are independent geometric entities. In four spacetime dimensions, an affine connection $\Gamma_{\mu\nu}^\lambda$ may have up to 64 independent components. Its antisymmetric part is known as the torsion tensor $S_{\mu\nu}^\lambda = (\Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda)/2$, which represents 24 degrees of freedom, while its symmetric part $C_{\mu\nu}^\lambda = (\Gamma_{\mu\nu}^\lambda + \Gamma_{\nu\mu}^\lambda)/2$ represents the remaining 40 degrees of freedom. When a metric is considered, the nonmetricity tensor $Q_{\alpha\mu\nu} = \nabla_\alpha^\Gamma g_{\mu\nu}$ can be used together with the torsion and the metric to fully determine the symmetric part of the connection as

$$C_{\mu\nu}^\lambda = L_{\mu\nu}^\lambda + g^{\lambda\alpha} g_{\rho\nu} S_{\mu\alpha}^\rho + g^{\lambda\alpha} g_{\rho\mu} S_{\nu\alpha}^\rho + \frac{g^{\lambda\alpha}}{2} (Q_{\mu\alpha\nu} + Q_{\nu\alpha\mu} - Q_{\alpha\mu\nu}) , \quad (1)$$

where $L_{\mu\nu}^\lambda$ represents the Levi-Civita connection of the metric, namely

$$L_{\mu\nu}^\lambda = \frac{g^{\lambda\alpha}}{2} (\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu}) . \quad (2)$$

We note that most of the literature on metric-affine gravity has focused on Einstein-Cartan spaces (with torsion but no nonmetricity [38]). In our case, however, we will consider torsionless scenarios with nonmetricity (Palatini approach). As we will see, nonmetricity arises naturally when one considers gravity Lagrangians beyond Einstein's theory.

A. $f(R)$ theories

We now consider extensions of Einstein's theory of the $f(R)$ type within the Palatini approach. We will omit details about the derivation of the field equations, as they are available in the literature (see for instance [39]). We just need to know that variation with respect to metric and connection lead, respectively, to

$$f_R R_{\mu\nu}(\Gamma) - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (3)$$

$$\nabla_\alpha [\sqrt{-g} f_R g^{\mu\nu}] = 0, \quad (4)$$

where $f_R \equiv df/dR$. Tracing the first of these equations with the metric $g^{\mu\nu}$ one obtains

$$Rf_R - 2f = \kappa^2 T. \quad (5)$$

This is an algebraic equation that relates R with the trace of the matter stress-energy tensor, T . In the case of GR, that relation is linear, $R = -\kappa^2 T$, but for nonlinear Lagrangians the relation will be, in general, nonlinear. In any case, note that R can be written as $R = R(T)$, implying that any function of R will be a function of T . As a result, the connection equation (4) can be seen as algebraic and linear for $\Gamma_{\mu\nu}^\alpha$ because the R dependences can be traded by functions of T . One thus finds that introducing an auxiliary metric, $h_{\mu\nu} = f_R g_{\mu\nu}$, Eq. (4) turns into $\nabla_\alpha [\sqrt{-h} h^{\mu\nu}] = 0$. This equation is equivalent to $\nabla_\alpha h_{\mu\nu} = 0$, whose solution is the well known Levi-Civita connection of $h_{\mu\nu}$

$$\Gamma_{\mu\nu}^\lambda = \frac{h^{\lambda\alpha}}{2} (\partial_\mu h_{\alpha\nu} + \partial_\nu h_{\alpha\mu} - \partial_\alpha h_{\mu\nu}). \quad (6)$$

By writing explicitly the dependence on $g_{\mu\nu}$ in this equation, one obtains the result advanced in Eq. (1) with $Q_{\alpha\mu\nu} = \nabla_\alpha^\Gamma g_{\mu\nu} = -(\partial_\alpha \ln f_R) g_{\mu\nu}$. The metric field equations can then be written in compact form as

$$R^\mu{}_\nu(h) = \frac{1}{f_R^2} \left(\frac{f}{2} \delta_\nu^\mu + \kappa^2 T^\mu{}_\nu \right), \quad (7)$$

where $R^\mu{}_\nu(h) = h^{\mu\alpha} R_{\alpha\nu}(h)$ and $R_{\alpha\nu}(h) \equiv R_{\alpha\nu}(\Gamma)$. Note that the R -dependent functions on the right-hand side of this equation are functions of the matter via the trace T . With these equations, one could solve for $h_{\mu\nu}$ once the matter sources are specified and then use the relation $h_{\mu\nu} = f_R g_{\mu\nu}$ to obtain $g_{\mu\nu}$. This strategy, however, is not always straightforward. One could thus opt in writing $R^\mu{}_\nu(h)$ in terms of $R^\mu{}_\nu(g)$ and get a set of equations directly referred to $g_{\mu\nu}$. Thus, Eq. (7) turns into

$$G_{\mu\nu}(g) = \kappa^2 \tau_{\mu\nu}. \quad (8)$$

where the modified (effective) stress-energy tensor, $\tau_{\mu\nu}$, is defined as

$$\begin{aligned} \kappa^2 \tau_{\mu\nu} = & \frac{\kappa^2}{f_R} T_{\mu\nu} - \frac{Rf_R - f}{2f_R} g_{\mu\nu} \\ & - \frac{3}{2f_R^2} \left[\partial_\mu f_R \partial_\nu f_R - \frac{1}{2} g_{\mu\nu} (\partial f_R)^2 \right] \\ & + \frac{1}{f_R} [\nabla_\mu \nabla_\nu f_R - g_{\mu\nu} \square f_R]. \end{aligned} \quad (9)$$

Written in this form, these equations can be interpreted as in Einstein's theory but with the right-hand side of Eq. (8) containing the stress-energy tensor $T_{\mu\nu}$ plus other functions which depend on the trace of the matter via $R(T)$. It is precisely this form of the equations that we will use to reinterpret our $f(R)$ wormhole solutions as solutions of Einstein's theory with a modified matter source. In other words, it is important to bear in mind that the modified stress-energy tensor $\tau_{\mu\nu}$ contains extra contributions which are just functions of the matter.

B. Beyond the $f(R)$ case

It has been recently established [40] that Palatini theories in which the gravity Lagrangian is a function of the Ricci tensor and the metric, $\mathcal{L}_G = f[g_{\alpha\beta}, R_{\mu\nu}(\Gamma)]$, lead to field equations of the form

$$R^\mu{}_\nu(h) = \frac{\kappa^2}{|\hat{\Omega}|^{1/2}} (\mathcal{L}_G \delta_\nu^\mu + T^\mu{}_\nu), \quad (10)$$

where $|\hat{\Omega}|$ represents the determinant of a matrix which relates the physical metric $g_{\mu\nu}$ and an auxiliary metric $h_{\mu\nu}$ which is compatible with the affine connection that defines the Ricci tensor. In other words, $h_{\mu\nu} = \Omega_\mu{}^\alpha g_{\alpha\nu}$ and $\nabla_\alpha^\Gamma h_{\mu\nu} = 0$.

In these theories, the nonmetricity tensor takes the form $Q_{\alpha\mu\nu} = g_{\rho\nu} (\nabla_\alpha \Omega_\mu{}^\lambda) \Omega_\lambda{}^\rho$ (symmetrized over μ and ν), where $\Omega_\mu{}^\alpha$ is a function of $T_{\mu\nu}$. In the $f(R)$ case, one has that $\Omega_\mu{}^\alpha = f_R \delta_\mu^\alpha$ (conformal transformation), which leads to a simple nonmetricity tensor. In general, however, the relation between the metrics is not conformal and a *clean* representation of the field equations in terms of $g_{\mu\nu}$ is not immediate. For this reason, Eq. (10) is typically preferred. Nonetheless, for the spherically symmetric scenarios that will be considered here, the Einstein tensor $G_{\mu\nu}(g)$ can always be computed straightforwardly, which facilitates the interpretation of the solutions in terms of an effective $\tau_{\mu\nu}$ in much the same way as in $f(R)$ theories.

III. GENERAL THEORY OF WORMHOLES

In 1955, John Wheeler introduced the term *geon* to denote a hypothetical gravitational-electromagnetic object with a nontrivial topological structure [7], and later with Misner [41] coined such a construction a *wormhole*. These concepts extended the very first calculations tracing back to the Einstein and Rosen bridge [8]. But it was the seminal work of Morris and Thorne [4] in 1988 which gave rise to the great interest on traversable Lorentzian wormholes which continues today. In this section, such concepts will be introduced, and the transition from wormholes in GR to those in metric-affine geometries will become clear.

A. Wormhole geometry

Lorentzian wormholes are nontrivial topological structures connecting two asymptotically flat regions, either in the same universe, or in different ones. The geometry of a static and spherically symmetric traversable (without event horizon) wormhole spacetime can be written as [4, 5]

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{1}{1-b(r)/r} dr^2 + r^2 d\Omega^2, \quad (11)$$

where (t, r, θ, ϕ) are the standard Schwarzschild coordinates, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ denotes the spherical sector. $\Phi(r)$ is called the redshift function because it is related to the gravitational redshift, and $b(r)$ is called the shape function as it determines the shape of the wormhole [4]. In order to describe the wormhole solution, two charts are needed to cover the upper/lower spacetimes, each of them with $r \in [r_0, +\infty)$, where r_0 is the radius of the surface where the two charts are joined and defines the throat of the wormhole. Consequently, one should explicitly write two different line elements for each chart with $\Phi_{\pm}(r)$ and $b_{\pm}(r)$ as the corresponding redshift and shape functions. However, we drop the \pm subscripts from now on in order not to overload the notation.

The throat connects two asymptotically flat regions and it is defined, for static configurations, as a surface of minimum area that satisfies the flaring-out condition, which can be obtained from the embedding calculations for the wormhole geometry and reads

$$\frac{b(r) - b'(r)r}{b^2(r)} > 0. \quad (12)$$

One thus deduces that $b'(r_0) < 1$ must hold, since at the throat $b(r_0) = r_0$ is imposed. Of course, the functions $\Phi(r)$ and $b(r)$ must fulfil some properties in order to describe a wormhole spacetime. The asymptotically flat limit imposes the restriction $b(r)/r \rightarrow 0$, as $r \rightarrow \infty$. Moreover, for the wormhole to be traversable there should be no event horizons so that $\Phi(r)$ must be finite everywhere. Note that it is straightforward to define the mass of the wormhole as the finite limit for the function $b(r)$. By comparison with the Schwarzschild geometry, an observer from spatial infinity sees that $b = 2G_N M$. It is worthwhile to mention that the mass of the wormhole could be different depending on the spatial location of the observer, although in this work we shall assume symmetric spacetimes on both sides of the wormhole throat.

The line element of a wormhole geometry can be alternatively written as

$$ds^2 = -e^{2\Phi(x)} dt^2 + dx^2 + r^2(x) d\Omega^2, \quad (13)$$

where the radial coordinate x is interpreted as the proper distance from the throat, located at $x = 0$, and related to r by $dx/dr = \pm 1/\sqrt{1 - b(r)/r}$. The advantage of this new set of coordinates lies in that it allows to describe the whole wormhole geometry by using a unique chart with $x \in (-\infty, +\infty)$. In terms of x , then it is clear that the area of the spherical surfaces never drops below $4\pi r_0^2$.

A useful way to construct wormhole spacetimes resides in the cut-and-paste procedure, which is a mathematical construction where two spacetimes are matched at a given junction interface. If this hypersurface contains non-zero surface stresses it is called a *thin-shell*, otherwise it is just a boundary surface. Following the standard junction-condition formalism [42], one cuts and pastes two manifolds to form a geodesically complete one

with the throat located at the joining shell, where the exotic matter is located [19] (see also Refs. [20] for a complete general analysis of spherically symmetric thin-shell wormholes). Beyond GR, the junction formalism requires to be generalized for the specific theory of gravity under consideration, for example, such as in metric $f(R)$ gravity [43].

At this point we would like to point out a special feature of the thin-shell structure. It is well known that thin-shell wormholes are geometric constructions that turn out to be geodesically complete although the Riemann tensor is divergent at the thin-shell where the throat is located [44]. To see this let Σ be a non-null hypersurface layer, and assuming the system of coordinates to be the same on both sides then \mathcal{X}_{\pm}^{μ} defines the jump of a quantity Z as

$$[Z] = Z(\mathcal{X}_{\mu}^{+})|_{\Sigma} - Z(\mathcal{X}_{\mu}^{-})|_{\Sigma}. \quad (14)$$

Then, the distribution of matter reads

$$T_{\mu\nu} = \Theta(x)T_{\mu\nu}^{+} + \Theta(-x)T_{\mu\nu}^{-} + T_{\mu\nu}|_{\Sigma}, \quad (15)$$

(where $\Theta(x)$ is Heaviside step function) so that the geodesics cross Σ when $x = 0$, and $T_{\mu\nu} = \delta(x)S_{\mu\nu}$, where $S_{\mu\nu}$ is the surface stress-energy tensor on the thin-shell.

This way, in the context of thin-shell wormholes the curvature of spacetime becomes divergent at Σ (because the Riemann tensor is singular) but this divergence is physically interpreted as a surface layer with a stress-energy tensor $T_{\mu\nu}|_{\Sigma}$ on it. Therefore, the existence of curvature divergences at the wormhole throat should not be surprising at all.

B. Energy conditions

Since the foundation of GR, a number of standard energy conditions [5], which are assumptions about the energy-matter content representing physically realistic situations found in Nature, have been thoroughly studied and classified in the literature. Assuming a stress-energy tensor of the form

$$T^{\mu}_{\nu} = \text{diag}(-\rho, p_1, p_2, p_3), \quad (16)$$

these conditions read:

- The weak energy condition (WEC) states that the energy density measured by an arbitrary observer must be non-negative, $\rho \geq 0$ and $\rho + p_i > 0$.
- The strong energy condition (SEC) asserts that gravity is attractive, $\rho + \sum p_i \geq 0$ and $\rho + p_i \geq 0$. Note that this condition is violated in many current models of accelerated cosmic expansion as well as in inflationary models.
- The dominant energy condition (DEC) expresses that the energy density measured by any observer

is positive but also that its flux propagates in a causal way (i.e., it is null or timelike) so that $\rho \geq 0$ and $\rho \geq |p_i|$.

- The null energy condition (NEC) implies that $\rho + p_i \geq 0$, $i = 1, 2, 3$.

Since $\text{WEC or SEC} \Rightarrow \text{NEC}$ and $\text{DEC} \Rightarrow \text{WEC} \Rightarrow \text{NEC}$, all energy conditions are automatically violated if the NEC is not accomplished. In particular, due to the flaring-out condition, wormhole spacetimes violate the NEC at the throat itself, and consequently violate all of the above pointwise energy conditions. Quantum effects can also produce violations of the classical energy conditions, which in order to support microscopic wormholes must typically be of order $\sim \hbar$. It is worth mentioning that there exist averaged versions of the energy conditions since quantum violations are probably not allowed globally (see [5] for more details).

Note that the energy conditions, in GR, can be traced back to the Raychaudhuri equation, where it is straightforward to determine that the attractive nature of gravity is represented by the condition that the Ricci tensor fulfils $R_{\mu\nu}k^\mu k^\nu \geq 0$ for any null vector k^μ . This condition ensures the focusing of the geodesic congruence, which in turn can be written, via Einstein's field equations, as a condition over the stress-energy tensor $T_{\mu\nu}k^\mu k^\nu \geq 0$ [45]. On the contrary, the wormhole structure requires that the null geodesic congruence must be scattered at the throat so as for geodesic completeness to hold, and this is precisely the physical meaning encoded in the flaring-out condition.

In extended theories of gravity, it is possible to express the gravitational field equations as in Eq. (8) where the effective stress-energy tensor $\tau_{\mu\nu}$ includes all new theory-dependent terms as well as the corresponding stress-energy tensor of the matter [46]. Thus, if one has repulsive gravity $R_{\mu\nu}k^\mu k^\nu < 0$, this implies $\tau_{\mu\nu}k^\mu k^\nu < 0$, but the matter stress-energy tensor can, in principle, be imposed to obey the energy conditions, or more specifically, in this case the NEC, i.e., $T_{\mu\nu}k^\mu k^\nu \geq 0$. Thus, in the context of modified gravity it has been shown that wormhole geometries are in fact supported by the effective stress-energy tensor, which actually plays the role of exotic matter (that could be interpreted as a gravitational fluid) and violates the energy conditions, while the physical matter satisfies them [21].

IV. WORMHOLES IN BORN-INFELD AND QUADRATIC PALATINI GRAVITY

We will now study a family of wormholes which arises as an exact solution of an extension of GR containing quadratic curvature scalar and Ricci-squared terms [47], and curiously also of Born-Infeld gravity [36], a theory that has attracted a good deal of attention in astrophysics and cosmology in the last few years [48]. Quadratic extensions of GR are supported by the quantization of

fields in curved spacetimes [49], by unifying approaches to quantum gravity [50], from effective Lagrangians methods [51], and are also employed as phenomenological tools to address issues like singularities, early cosmology, astrophysics, and alternatives to the dark matter/energy paradigm [see e.g. [52] for some reviews]. The action of Born-Infeld gravity reads

$$S = \frac{1}{\kappa^2 \epsilon} \int d^4x \left[\sqrt{-|g_{\mu\nu} + \epsilon R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{-|g_{\mu\nu}|} \right] + S_m[g_{\mu\nu}, \psi], \quad (17)$$

where a vertical bar denotes a determinant, $\epsilon = -2l_\epsilon^2$ is a small parameter with dimensions of length squared and assumed to be negative ($l_\epsilon^2 > 0$), the choice of the constant $\lambda = 1$ sets an asymptotically flat spacetime and S_m is the matter action. A series expansion of this action in the parameter ϵ recovers GR with a cosmological constant term to leading order, and a quadratic Lagrangian $\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}$ at next-to-leading order.

In Sec. III, we presented the equations that theories like this Born-Infeld model satisfy [see Eq.(10)]. For static and spherically symmetric scenarios, the metric and the deformation matrix Ω_μ^α are diagonal, and exact solutions for electrovacuum configurations can be found analytically (see Refs. [36, 47] for details). Here we will use those solutions to interpret them from the prism of GR, i.e., using them we compute its associated Einstein tensor and interpret it as an effective stress-energy tensor. The properties of such effective matter source will be discussed.

A. Wormhole geometry

The line element for these electrically charged objects takes the form [36]

$$ds^2 = -A(x)dt^2 + \frac{1}{A(x)\Omega_+^2}dx^2 + (r_c z)^2(x)d\Omega^2, \quad (18)$$

with the following definitions: $z \equiv r/r_c$ is a dimensionless radial function with $r_c = \sqrt{r_q l_\epsilon}$, and $r_q^2 = 2G_N q^2$ is a charge scale parameterizing the solutions, the objects $\Omega_\pm = 1 \pm 1/z^4$, and the metric function $A(x)$ is given by

$$A(x) = \frac{1}{\Omega_+} \left[1 - \frac{(1 + \delta_1 G(z))}{\delta_2 z \Omega_-^{1/2}} \right], \quad (19)$$

and the two constants parameterizing the metric are obtained as

$$\delta_1 = \frac{1}{2r_S} \sqrt{\frac{r_q^3}{l_\epsilon}}; \quad \delta_2 = \frac{r_c}{r_S}, \quad (20)$$

where $r_S = 2G_N M_0$ is the Schwarzschild radius.

Finally, the function $G(z)$, which satisfies the equation

$$\frac{dG}{dz} = \frac{\Omega_+}{z^2 \Omega_-^{1/2}}, \quad (21)$$

can be analytically integrated and expressed as an infinite power series of the form

$$G(z) = -\frac{1}{\delta_c} + \frac{1}{2}\sqrt{z^4 - 1} [f_{3/4}(z) + f_{7/4}(z)] , \quad (22)$$

where $f_\lambda(z) = {}_2F_1[\frac{1}{2}, \lambda, \frac{3}{2}, 1 - z^4]$ is a hypergeometric function, and $\delta_c \approx 0.572069$ is an integration constant necessary to get the correct asymptotic behavior of GR. Since the radial function $r(x)$ in Eq. (18) is defined by

$$x^2 = \Omega_- r^2 \rightarrow \frac{dx}{dz} = \pm \frac{\Omega_+}{\Omega_-^{1/2}} , \quad (23)$$

we will be interested in analyzing the behaviour in two particular limits: far from ($z \gg 1$) and near the region ($z \approx 1$), which respectively means $r \gg r_c$ and $r \approx r_c$. Moreover, the radial function $r(x)$ defined above can be explicitly written as

$$r^2(x) = \frac{x^2 + \sqrt{x^4 + 4r_c^4}}{2} \quad (24)$$

which is a key element in the characterization of these solutions.

The geometry encoded in the line element (18) describes a spacetime which looks much like the Reissner-Nordström (RN) solution of the Einstein-Maxwell field equations of GR as long as $r \gg r_c$. This follows from the expansion of $G(z) \simeq -1/z$ for $z \gg 1$, in such a way that $\Omega_\pm \simeq 1$ and

$$A(x) \approx 1 - \frac{r_s}{r} + \frac{r_q^2}{2r^2} . \quad (25)$$

It should be noted that the smallness of the corrections induced by the Ω_\pm functions implies that the standard GR description is valid all the way down to $r \gtrsim 2r_c$, with noticeable deviations with respect to the RN solution found only within the region $r \lesssim 2r_c$. With more detail, the metric function at $r = r_c$ behaves as

$$\lim_{r \rightarrow r_c} A(x) \approx \frac{N_q}{4N_c^\epsilon} \frac{(\delta_1 - \delta_c)}{\delta_1 \delta_c} \sqrt{\frac{r_c}{r - r_c}} + \frac{1}{2} \left(1 - \frac{N_q}{N_c^\epsilon} \right) + O(\sqrt{r - r_c}) , \quad (26)$$

where we have defined the number of charges as $N_q = q/e$, being e the proton charge, and introduced the constant $N_c^\epsilon \equiv N_c l_\epsilon / l_P$, where $l_P = \sqrt{\hbar G / c^3}$ is the Planck length and $N_c \equiv \sqrt{2/\alpha_{em}} \approx 16.55$ is a constant (with α_{em} the fine structure constant), which will play an important role later. This expansion manifests a strong deviation with respect to the GR behavior. In this region, one finds that the radial function $r(x)$ defined by Eq. (24) cannot become smaller than r_c (corresponding to $x = 0$), where it bounces off [see Eq. (24)] to another region of spacetime [see Fig. 1]. Thus, the innermost region of these electrically charged objects, which in GR ends up in a point-like singularity at $r = 0$, now becomes replaced by a wormhole structure for all values of the

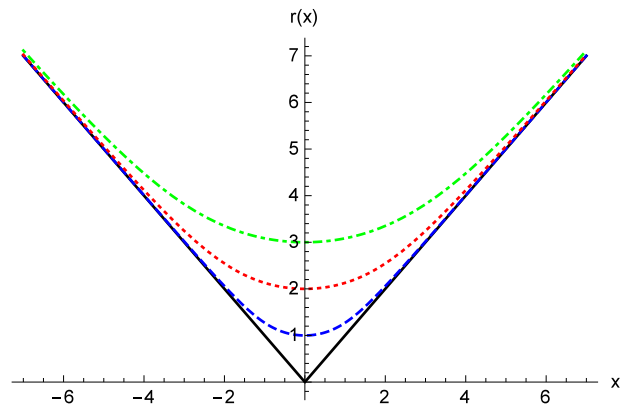


Figure 1. Representation of the radial function $r(x)$ (for $r_c = 1, 2, 3$ dashed blue, dotted red and dotted-dashed green, respectively), measured in units of r_c . The wormhole throat is located at $x = 0$, where the area of the 2-spheres reaches a minimum and bounces off to another region of spacetime. Note that a few r_c units away from the wormhole throat the radial functions quickly converges to the behaviour of the standard Reissner-Nordström solution, $r \approx |x|$ (straight solid black line).

charge-to-mass ratio δ_1 , with a spherical throat of area $4\pi r_c^2$.

Whether the wormhole is covered by a horizon or not depends on the charge-to-mass ratio δ_1 , yielding three different structures [36, 47]

- $\delta_1 > \delta_c$: This is essentially similar to the standard Reissner-Nordström solution of GR, in that there may be two, one (degenerate) or no horizons on each side of the wormhole.
- $\delta_1 < \delta_c$: A single (non-degenerate) horizon always exists on each side of the wormhole, resembling the Schwarzschild black hole.
- $\delta_1 = \delta_c$: This configuration exhibits a single horizon (on each side) which disappears if $N_q < N_c^\epsilon$. For these configurations the metric at $r = r_c$ is finite (i.e. Minkowskian-like).

It should be noted that for macroscopic objects ($N_q \gg N_c^\epsilon$) the location of the event horizon is essentially the same as in the Reissner-Nordström solution of GR, $r_h \approx (r_s + \sqrt{r_s^2 - r_q^2})/2$. Only for microscopic solutions ($N_q \sim N_c^\epsilon$) do we find the qualitative differences of the above classification.

B. Energy conditions

Let us now assume that the line element (18) is a solution of Einstein's equations coupled to a certain fluid: $G_{\mu\nu}(g) = \kappa^2 \tau_{\mu\nu}$, recalling that $\tau_{\mu\nu}$ is the effective stress-energy tensor. As discussed in Sec.II, this can always be done explicitly, and one finds that the specific form of

$\tau_{\mu\nu}$ depends both on the matter source and the theory of gravity (which can also be written as a function of the matter) chosen. As emphasized above, in such a reinterpretation of the theory, it should not come as a surprise the existence of matter-energy sources in which $T_{\mu\nu}$ satisfies the energy conditions, while $\tau_{\mu\nu}$ does not [46]. Let us apply such a procedure to our spacetime metric (18).

We shall assume that the components of the effective stress-energy tensor can be written as

$$\tau^\mu{}_\nu = \begin{pmatrix} -\rho_{\text{eff}} & 0 & 0 & 0 \\ 0 & -\tau_{\text{eff}} & 0 & 0 \\ 0 & 0 & p_{\text{eff}} & 0 \\ 0 & 0 & 0 & p_{\text{eff}} \end{pmatrix}, \quad (27)$$

where ρ_{eff} is the effective energy density, τ_{eff} is the effective radial tension, and p_{eff} are the effective transverse pressures. Note that this carries the implicit assumption that the effective stress-energy tensor can be cast in a diagonal form, which is reasonable (and can be explicitly verified) given the fact that the extra matter-energy sources appearing in the corresponding field equations are functions of the (diagonal) stress-energy tensor of the electromagnetic field (see Eq. (9) as an example).

In order to follow the standard representation of the wormhole geometry for static spherically symmetric spacetime [4, 5], we first use the change of coordinates $dx/dz = \Omega_+/\Omega_-^{1/2}$ in Eq. (23) to rewrite the line element (18) in a more canonical form as

$$ds^2 = -A(x)dt^2 + \frac{1}{A(x)\Omega_-}dz^2 + z^2(x)d\Omega^2. \quad (28)$$

In the standard representation of wormhole geometries the radial coordinate $z^2 = z^2(x)$ is assumed to bounce off at (say) $x = 0$. In our case, this condition is not imposed but, rather, it is naturally achieved by virtue of the geometry described in the previous section and, in particular, by Eq. (24), which emerges directly from the resolution of the corresponding field equations. Note that the use of $z(x)$ as a radial coordinate is subjected to the important restriction that this is only valid in those regions where $z(x)$ is a monotonic function. Therefore, in agreement with the standard procedure of wormhole physics, one needs two charts (copies) of the coordinate $z(x)$ to cover the whole manifold, but a single one in terms of x .

Comparing the line element (28) of the Born-Infeld gravity black holes, with that of the standard wormhole one, Eq. (11), one simply needs to introduce the identifications

$$\tilde{\Phi} = \frac{1}{2} \log(A(x)), \quad (29)$$

$$\tilde{b}(z) = z(1 - A(x)\Omega_-), \quad (30)$$

where we have defined dimensionless variables $\tilde{b}(z) = b(z)/r_c$ and $\tilde{\Phi}(z) = \Phi(z)/r_c$, for coherency in the notation. Now, one just needs to replace the stress-energy

tensor (27) into the Einstein field equations to obtain

$$\tilde{\rho} \equiv r_c^2 \rho_{\text{eff}} = \frac{d\tilde{b}/dz}{z^2}, \quad (31)$$

$$\tilde{\tau} \equiv r_c^2 \tau_{\text{eff}} = \frac{\tilde{b}(z)}{z^3} - 2 \left(1 - \frac{\tilde{b}(z)}{z} \right) \frac{d\tilde{\Phi}/dz}{z}, \quad (32)$$

$$\tilde{p} \equiv r_c^2 p_{\text{eff}} = \left(1 - \frac{\tilde{b}(z)}{z} \right) \left[\frac{d^2\tilde{\Phi}}{dz^2} + \frac{d\tilde{\Phi}}{dz} \left(\frac{d\tilde{\Phi}}{dz} + \frac{1}{z} \right) \right] - \frac{1}{2z^2} \left(\frac{d\tilde{b}}{dz} z - \tilde{b}(z) \right) \left(\frac{d\tilde{\Phi}}{dz} + \frac{1}{z} \right), \quad (33)$$

and the framework is ready to study the matter source threading these wormholes in the context of GR.

Among the spectrum of solutions of our theory, let us just focus on those corresponding to *traversable* wormholes, for which no horizon is present¹. This is the case where the representation given by Eq. (11) of the wormhole geometry is valid. In our case, traversable wormholes correspond to i) solutions with $\delta_1 = \delta_c$ and $N_q < N_c^\epsilon$, and ii) solutions with $\delta_1 > \delta_c$ and no horizons (naked configurations).

This way, we insert the spacetime metric (18) into the set of equations (31)-(33) with the definitions above, and solve them to obtain the components of the effective stress-energy tensor (27) defining the matter source. One then obtains involved expressions for all such components but, nonetheless, we are just interested in exact expressions in the far limit, $r \gg r_c$ (equivalently, $z \gg 1$ or $x \gg 0$), and close to the center, $r = r_c$ (equivalently, $z = 1$ or $x = 0$). At far distances, one gets

$$\tilde{\rho}(r \gg r_c) = \tilde{\tau}(r \gg r_c) = \tilde{p}(r \gg r_c) \simeq \frac{N_q}{N_c^\epsilon x^4}. \quad (34)$$

Reverting the notation, this is just the familiar result

$$\rho(r \gg r_c) = \tau(r \gg r_c) = p(r \gg r_c) \simeq \frac{q^2}{r^4}, \quad (35)$$

which is nothing but the energy, tension and pressure of the Maxwell field supporting the standard Reissner-Nordström solution, in agreement with the recovery of this solution in the far limit.

¹ It is worth pointing out the existence of a potential confusion regarding terminology. If the word *traversable* is employed instead to indicate that an observer can cross the wormhole throat, this question has been positively established in our case using an analysis of the geodesic completeness on these spacetimes [53, 54] and of the evolution of the volume element associated to a physical observer crossing the wormhole throat [55]. This result is in agreement with the well posed nature of the problem of scattering of a scalar wave off the wormhole [53]. This way, traversable as equivalent to *safe crossing* is guaranteed for all these spacetimes, having horizons or not. Nonetheless, to be coherent with the standard nomenclature on the field, we will stick ourselves to the idea of traversable as *no horizons*.

Close to the spherical surface $r = r_c$ (or $z = 1$), where $x = 0$ (see Eq.(24)), the expansion of the components of the stress-energy tensor yields the result

$$\lim_{x \rightarrow 0} \tilde{\rho} \approx -\frac{N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1 |x|} + \left(\frac{2N_q}{N_c^\epsilon} - 1 \right) + \mathcal{O}(x^2), \quad (36)$$

$$\lim_{x \rightarrow 0} \tilde{\tau} \approx \frac{N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1 |x|} + 1 - \frac{2N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1} |x| + \mathcal{O}(x^2), \quad (37)$$

$$\lim_{x \rightarrow 0} \tilde{p} \approx \frac{2N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1 |x|^3} - \frac{N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1 |x|} + \left(2 - \frac{5N_q}{3N_c^\epsilon} \right) + \mathcal{O}(x^2). \quad (38)$$

To further interpret these results let us bring to our previous discussion the standard classification of the energy conditions. From among the four (null, weak, strong and dominant) we are mostly interested in the NEC, because its violation implies the violation of all the others (recall the discussion of Sec.IV). This condition states that, for any null vector k^μ , the inequality $T_{\mu\nu} k^\mu k^\nu \geq 0$ must be satisfied. Assuming the matter stress-energy tensor to have the form given by Eq. (16), one obtains

$$\tilde{\rho} + \tilde{p}_j \geq 0, \quad \forall j = 1, 2, 3, \quad (39)$$

where $\tilde{p}_1 = -\tilde{\tau}$ and $\tilde{p}_2 = \tilde{p}_3 = \tilde{p}$. To study this condition, we observe that around the wormhole throat:

$$\lim_{x \rightarrow 0} (\tilde{\rho} - \tilde{\tau}) \simeq -\frac{2N_q(\delta_1 - \delta_c)}{N_c^\epsilon \delta_c \delta_1 |x|} + 2 \left(\frac{N_q}{N_c^\epsilon} - 1 \right) + \mathcal{O}(x), \quad (40)$$

and thus it is clear that for horizonless remnants, $\delta_1 = \delta_c$ and $N_q \leq N_c^\epsilon$, the first term is zero and the leading order term becomes negative at the wormhole throat, $\lim_{x \rightarrow 0} (\tilde{\rho} - \tilde{\tau}) < 0$. As depicted in Fig. 2, this means that the NEC is violated in a small region around $x = 0$. The fact that only exotic matter can support these geometries is in complete agreement with the standard wisdom of wormholes within GR. Note in passing by that whether the energy density becomes negative, $\tilde{\rho} < 0$, or not, depends on additional constraints upon the number of charges N_q , as can be seen from the comparison between Figs. 2 and 3.

For configurations with $\delta_1 > \delta_c$ the existence or absence of horizons depends on the charge-to-mass ratio, which should be large (more charge than mass in certain units). Nonetheless, it is clear from Eqs. (36) and (40) that the NEC will be violated because $\lim_{x \rightarrow 0} (\tilde{\rho} - \tilde{\tau}) < 0$ (and one always has $\lim_{x \rightarrow 0} \tilde{\rho} < 0$). When the GR solutions are a good approximation, the absence of horizons translates into the condition $2r_q^2/r_S^2 > 1$ which, knowing that $\delta_1 = (N_q/N_c^\epsilon)\delta_2$, can be written as $(\delta_1/\delta_c) \gtrsim \sqrt{N_q/N_c^\epsilon}$. In Fig. 4, we plot a particular case with $\delta_1 = \delta_c + 0.3$ and $N_q = N_c + 10$, which satisfies the above no horizons constraint. There it is seen that the NEC is violated around the throat by an infinite amount, which is in contrast with the finite violation achieved in the $\delta_1 = \delta_c$ cases. By completeness, we also point out

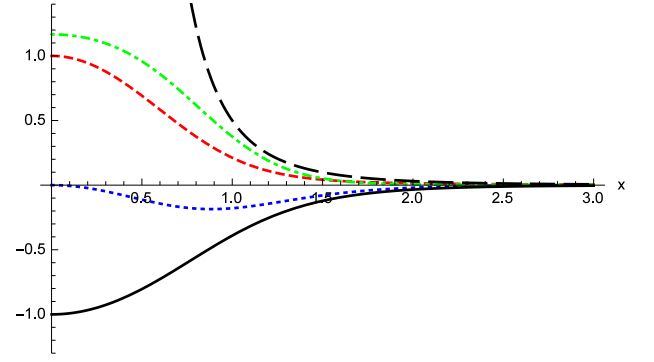


Figure 2. The components $\tilde{\rho}$ (blue dotted), $\tilde{\tau}$ (red thin dashed) and \tilde{p} (green dashed-dotted) of the stress-energy tensor (27) as a function of x for regular horizonless solutions ($\delta_1 = \delta_c$) with $N_q = N_c^\epsilon$, which represents the transition case where the horizon disappears. At large distances all these components reduce to those of the Reissner-Nordström case (black thick dashed, upper curve in this figure), $N_q/(N_c^\epsilon x^4)$ [see Eq. (34)]. The solid black line (lower curve in this figure) represents $[\tilde{\rho} - \tilde{\tau}] < 0$, which means that the NEC is violated near the wormhole throat.

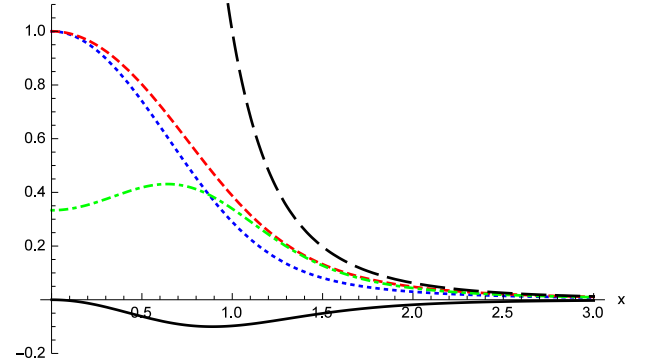


Figure 3. Same notation and curves as in Fig. 2, now with $N_q = N_c^\epsilon/2$ (for $l_e = l_P$). Note that the NEC is violated both due to $\tilde{\rho} < 0$ (blue dotted) and $[\tilde{\rho} - \tilde{\tau}] < 0$ (solid black).

that the flaring-out condition (12) can be numerically checked to be satisfied both in the $\delta_1 > \delta_c$ and $\delta_1 = \delta_c$ cases above.

From our analysis it is clear that the emergence of the wormhole structure from the prism of GR is related to the deviations that the stress-energy tensor suffers with respect to that of a Maxwell field as the innermost region $r = r_c$ is approached. This is so because the symmetry satisfied by the stress-energy tensor characterizing electromagnetic fields, namely, $T_t^t = T_r^r$ and $T_\theta^\theta = T_\varphi^\varphi$ cannot support wormhole solutions in the context of GR, even for non-linear electromagnetism [37]. Thus, one is forced to consider exotic sources of matter to generate the wormhole. Alternatively, one can re-consider this scenario and interpret these wormholes as emerging out of modified gravity effects, in which case electromagnetic fields (either Maxwell or non-linear [56]) with $T_t^t = T_r^r$ and $T_\theta^\theta = T_\varphi^\varphi$ but $\tau_t^t \neq \tau_r^r$ and $\tau_\theta^\theta \neq \tau_\varphi^\varphi$ can naturally

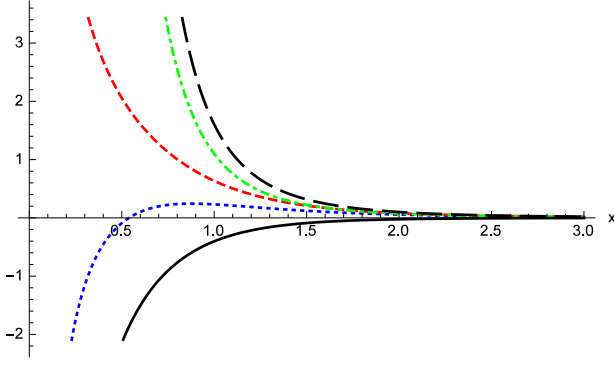


Figure 4. Same notation and curves as in Fig. 2, now with $\delta_1 = \delta_c + 0.3$ and $N_q = N_c^\epsilon + 10$, corresponding to Reissner-Nordström-like configurations without horizons (naked). Note that the NEC is violated because $\tilde{\rho} - \tilde{\tau} < 0$ (solid black line) around the wormhole throat $x = 0$.

sustain them without violation of the energy conditions.

It is worth noting that in all cases the typical size of the region where these violations of the energy conditions occur is determined by the scale $r_c = \sqrt{r_q l_\epsilon} = l_\epsilon \sqrt{2N_q/N_c^\epsilon}$ (observe that the x -scale and the vertical scale in Figs. 2–3 are both measured in units of r_c). This implies that violations of the energy conditions are restricted to a region of order $\sqrt{l_P l_\epsilon}$ which may grow with the charge as $N_q^{1/2}$ (recall that $N_c^\epsilon \approx 16.55 l_\epsilon / l_P$). Thus, unless l_ϵ be much greater than l_P and/or one considers huge amounts of charge, the size of this region will typically be very small.

Let us stress that, though curvature divergences may arise at the wormhole throat r_c , all these wormholes are geodesically complete for all spectra of mass and charge and physical observers can safely go through them [53]. This should not come as a surprise since the same feature is quite common in the thin-shell approach discussed in Sec. III A.

V. $f(R)$ WORMHOLES

For the discussion of this type of wormholes, we restrict ourselves to quadratic $f(R)$ models of the form

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R - \lambda R^2) + S_m(g_{\mu\nu}, \psi_m), \quad (41)$$

where λ is a constant with dimensions of length squared, and in which the matter sector is represented by an anisotropic fluid whose stress-energy tensor reads

$$T^\mu{}_\nu = \text{diag}(-\rho, -\rho, \alpha\rho, \alpha\rho), \quad (42)$$

where $0 \leq \alpha \leq 1$ is necessary to satisfy the energy conditions. This type of fluids has been recently considered in order to investigate the existence of wormhole solutions in several extensions of GR [29]. Note the case $\alpha = 1$ corresponds to a standard Maxwell field, which

has a vanishing trace for $T_\mu{}^\mu$. Because of the fact that only matter sources with a nonzero trace can excite the dynamics of Palatini $f(R)$ gravity (recall the discussion of section II A) we shall be concerned here with the cases $\alpha \neq 1$. Let us also point out that models of non-linear electrodynamics have the same structure (42) in their stress-energy tensor.

A. Wormhole geometry

The solutions of this gravity-matter system are characterized by the following line element, suitably adapted to our problem [35]:

$$ds^2 = -A(z)dt^2 + \frac{1}{A(z)f_R^3} \left(1 + \frac{\alpha}{z^{2+2\alpha}}\right)^2 dz^2 + z^2(x)d\Omega^2, \quad (43)$$

where $z = r/r_c$ is now defined in terms of $r_c^{2+2\alpha} = 1 - (4\lambda)\kappa^2(1-\alpha)C$ (C is a dimensional integration constant such that the energy density reads $\rho(x) = C/r(x)^{2+2\alpha}$) and the metric function can be obtained in closed analytical form as

$$A(z) = \left(1 - \frac{1 + \delta_1 G_\alpha(z)}{\delta_2 z f_R^{1/2}}\right) f_R^{-1}, \quad (44)$$

with

$$f_R = 1 - \frac{1}{z^{2+2\alpha}}, \quad (45)$$

where we have defined the following constants

$$\delta_1 = \frac{r_c^3}{8\lambda M_0}, \quad \delta_2 = \frac{r_c}{2M_0}, \quad (46)$$

and the function $G_\alpha(z)$ is given by

$$G_\alpha(z) = \frac{z^{-4\alpha-1}}{2(\alpha-1)(2\alpha-1)} \times \left[\frac{z^{2\alpha+2} \sqrt{1-z^{-2(\alpha+1)}} (2\alpha^2 + \alpha + 2z^{2\alpha+2} - 3)}{(z^{2\alpha+2} - 1) 2(\alpha-1)(2\alpha-1)} - \frac{8\alpha(\alpha^2 - 1) {}_2F_1\left(\frac{1}{2}, \frac{4\alpha+1}{2\alpha+2}; \frac{6\alpha+3}{2\alpha+2}; z^{-2(\alpha+1)}\right)}{(4\alpha+1)} \right], \quad (47)$$

where ${}_2F_1(a, b, c; y)$ is a hypergeometric function and M_0 is the Schwarzschild mass. Note that the expression for $G_\alpha(z)$ is valid as long as $\alpha \neq 1$ and $\alpha \neq 1/2$, in which cases different expressions are found [see Ref. [35] for details].

Like in the previous case, the wormhole structure is manifest from the behaviour of the radial function $r(x)$, which satisfies the equation

$$\frac{dz}{dx} = \frac{f_R^{1/2}}{\left(1 + \frac{\alpha}{z^{2+2\alpha}}\right)} \Rightarrow x = z \sqrt{1 - \frac{1}{z^{2+2\alpha}}}, \quad (48)$$

which could be used to rewrite the geometry (43) with x as the radial coordinate, similarly as in Eq. (18) of the Born-Infeld gravity wormholes. Far from the center, $z \gg 1$, one recovers $r \simeq x$, in agreement with the fact that the corrections to GR vanish there. Close to the center $z \approx 1$ (where $r = r_c$ or $x = 0$), one finds that the radial function reaches a minimum and bounces off, similarly to what was found in the previous section, though in this case a closed expression for $z(x)$ is not easy to obtain for generic α . Nonetheless, in Fig. 5 we have numerically integrated the relation (48), where we see the expected bouncing behavior.

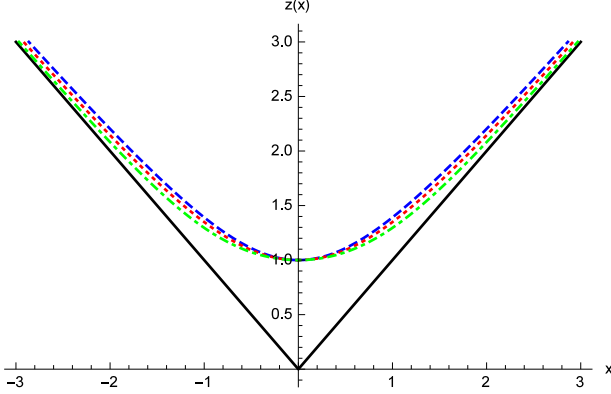


Figure 5. Behaviour of the radial function $z(x)$ in the $f(R)$ gravity case, obtained from Eq.(48), for the cases $\alpha = 1/10, 1/3, 3/4$ (dashed blue, dotted red and dotted-dashed green, respectively). At $x \gg 1$ the function $z^2(x) \simeq x^2$ and the GR behaviour is restored (straight solid black line). Observe the bouncing behaviour of all these curves at $x = 0$.

Following the definitions introduced above, the shape function $\tilde{b}(z)$ characterizing the wormhole is now written as

$$\tilde{b}(z) = z \left(1 - \frac{A(z)f_R^3}{\left(1 + \frac{\alpha}{z^{2+2\alpha}}\right)^2} \right), \quad (49)$$

while the redshift function $\tilde{\Phi}(z)$ is still formally given by Eq. (29), now with the set of definitions (44)–(47).

For arbitrary α , a series expansions both at $z \rightarrow \infty$ and as $z \simeq 1$ yield long expressions, which are of little use for our purposes. Therefore we shall consider a particular case, say $\alpha = 3/4$, to illustrate the analysis. First we note that the asymptotic behavior of the metric in this case becomes

$$A(z) \approx 1 - \frac{\delta_2}{z} + \frac{8\delta_1}{\delta_2 z^{3/2}} + \mathcal{O}\left(\frac{1}{z^{7/2}}\right), \quad (50)$$

whose fall off is governed by the standard Schwarzschild mass term δ_2/z , and is asymptotically flat. Note that the charge term contribution has been softened, from the $\sim 1/z^2$ contribution of the Reissner-Nordström space-time, to the $\sim 1/z^{3/2}$ of the present case. Indeed, this is a general property of all these models with $1/2 < \alpha < 1$,

whereas those with $0 < \alpha < 1/2$ are still asymptotically flat, but governed by the “charge” term instead of the Schwarzschild mass term. This is in agreement with the fact that the stress-energy tensor of the fluid given by Eq. (42) can also be reproduced by non-linear theories of electrodynamics with $L_m = X^\alpha$, where $X = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$, and those models with $0 < \alpha < 1/2$ have been found to be asymptotically flat but not Schwarzschild-like (*anomalous* models [57]). Note that, as opposed to the Maxwell case, $L_M = X$, the trace of such models is non-vanishing, which allows to excite the non-linear dynamics driven by the $f(R)$ gravity [since in these models $R = -\kappa^2 T$].

B. Energy conditions

Using Eq. (50) above, one can get the corresponding asymptotic behavior of the energy density and the pressures, namely

$$\tilde{\rho}(z \gg 1) \approx 4\frac{\delta_1}{\delta_2} \frac{1}{z^{7/2}} - \frac{35}{4} \frac{1}{z^{11/2}} + \mathcal{O}\left(\frac{1}{z^{13/2}}\right), \quad (51)$$

$$\tilde{\tau}(z \gg 1) \approx 4\frac{\delta_1}{\delta_2} \frac{1}{z^{7/2}} + 7\frac{1}{z^{11/2}} + \mathcal{O}\left(\frac{1}{z^{13/2}}\right), \quad (52)$$

$$\tilde{p}(z \gg 1) \approx 3\frac{\delta_1}{\delta_2} \frac{1}{z^{7/2}} + \frac{49}{4} \frac{1}{z^{11/2}} + \mathcal{O}\left(\frac{1}{z^{13/2}}\right). \quad (53)$$

To first order we recover the GR expressions, which satisfy the energy conditions. This contrasts with the higher-order corrections generated by the $f(R)$ dynamics, which violate the NEC as $\tilde{\rho}(z \gg 1) - \tilde{\tau}(z \gg 1) < 0$. From our definitions, one verifies that such corrections are highly suppressed by a factor $\sim (\lambda\kappa^2 C)^{11/7}/r^{11/2}$. Thus, for sufficiently small values of the length squared scale λ such asymptotic violations of the energy conditions would lie far beyond experimental reach.

In the opposite limit, near the wormhole (now we expand around the wormhole throat located at $z = 1$ instead of x since, as mentioned above, it is not possible to obtain a closed expression $z = z(x)$), we find

$$\tilde{\rho} \approx 1 - 4\frac{\delta_1}{\delta_2} + \frac{3\left(8 - \frac{21\sqrt{\pi}\Gamma[\frac{15}{7}]\delta_1}{\Gamma[\frac{23}{14}]}\right)}{2\sqrt{14}\delta_2}(z-1)^{1/2} + \mathcal{O}(z-1), \quad (54)$$

$$\tilde{\tau} \approx 1 + 8\frac{\delta_1}{\delta_2} - \frac{3\left(8 - \frac{21\sqrt{\pi}\Gamma[\frac{15}{7}]\delta_1}{\Gamma[\frac{23}{14}]}\right)}{2\sqrt{14}\delta_2}(z-1)^{1/2} + \mathcal{O}(z-1), \quad (55)$$

$$\tilde{p} \approx 6\frac{\delta_1}{\delta_2} \frac{1}{(z-1)} - \frac{3\left(8 - \frac{21\sqrt{\pi}\Gamma[\frac{15}{7}]\delta_1}{\Gamma[\frac{23}{14}]}\right)}{4\sqrt{14}\delta_2} \frac{1}{(z-1)^{1/2}} + \mathcal{O}(z-1), \quad (56)$$

where $\Gamma[a]$ is Euler’s gamma function. From these expressions it is immediately seen that $\lim_{z \rightarrow 1}(\tilde{\rho} - \tilde{\tau}) < 0$ and thus the NEC is violated regardless of the values of the constants δ_1 and δ_2 characterizing the solutions. In

addition, when $\delta_1/\delta_2 > 1/4$ one has that $\lim_{z \rightarrow 1} \tilde{\rho} < 0$. In Fig. 6, the different components of the stress-energy tensor are plotted for the case $\delta_2 = 4\delta_1$ and $\delta_1 = 10$, in which one can numerically check that no horizon is present and thus we are dealing with traversable wormholes. In this plot we observe a bounded violation of the null energy condition near the throat, similarly to what was found in the $\delta_1 = \delta_c$ configurations of the Born-Infeld gravity theory studied in the previous section. In addition, one can numerically check that the flaring-out condition (12) is satisfied for these wormhole geometries.

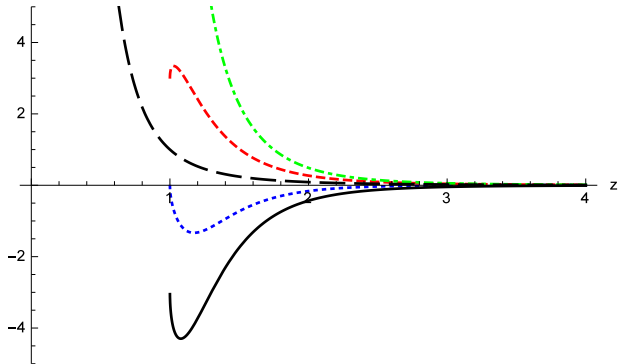


Figure 6. Same notation and curves as in Fig. 2, now for the $f(R)$ gravity case with $\alpha = 3/4$ and parameters $\delta_2 = 4\delta_1$ and $\delta_1 = 10$, for which no horizons are present. Note that the NEC is violated because $\lim_{x \rightarrow 0}(\tilde{\rho} - \tilde{\tau}) < 0$ (solid black) around the wormhole throat $z = 1$.

Let us stress that, like in the Born-Infeld gravity case, for the existence of these wormhole solutions it is not enough to have a non-linear electrodynamics source. This follows from the fact that in GR no wormholes supported by this kind of matter source are allowed [37]. Indeed, as follows from Eqs. (55)–(57) the *effective* stress-energy tensor that sources our wormholes does not have the symmetries of nonlinear theories of electrodynamics, namely, $\tau_t^t \neq \tau_x^x$ and $\tau_\theta^\theta \neq \tau_\varphi^\varphi$, due to the presence of λ -corrections. When λ vanishes, the wormhole throat closes and one recovers the GR results. Note that the preceding discussion could be further extended to include other theories of (non-linear) electrodynamics, where wormholes have been recently found [34].

VI. ENDING COMMENTS

In this work we have presented two classes of traversable wormhole spacetimes (that means without horizons) which are supported by a single matter source given in the action from which the geometry is derived and turns out to be well defined everywhere. These spacetimes are exact solutions of certain extensions of GR including higher-order powers and contractions of the Ricci tensor, and formulated in the Palatini approach. Here we have shown that they can be also interpreted as

exact solutions of GR with a modified or effective stress-energy tensor. We have obtained such effective stress-energy tensors and discussed the energy conditions. We stress that these solutions are not constructed as a result of a copy-and-paste procedure of two spacetimes, but instead directly arise directly from well defined actions.

From the GR viewpoint, we have found that the energy conditions for wormholes coming from Born-Infeld and $f(R)$ gravity are violated at/near the wormhole throat, which is in perfect agreement with the current knowledge of wormholes physics. Although the kind of matter source, needed to sustain a wormhole from this point of view, is quite peculiar, it is confined in a restricted region around the wormhole throat and more important it does not seem to be in contradiction with any fundamental physics principle. However, from the modified gravity side there is a straightforward way to reformulate the standard view by describing a wormhole spacetime actually threaded by ordinary matter but supported by the novel gravitational effects attached to the particular extended gravity and whose size is determined by a fundamental length scale, which in turn defines the size of the violations of the energy conditions from the GR viewpoint. Certainly, the properties or the behavior of matter could be dependent on the theoretical framework adopted. Therefore, it is an overriding issue to keep exploring alternative theories of gravity. The Palatini approach is indeed an encouraging new perspective by making possible to deal with the problem of singularities by replacing them with microscopic wormholes sustained by ordinary matter.

It should be stressed that the issue of geodesic completeness is not guaranteed *a priori* in our wormholes; in fact, it should be inspected *a posteriori* in order to guarantee the *safeness* of physical observers during their transit from one of the wormhole mouths to the other. It is worth mentioning that this feature is entirely different to the very well known case of thin-shell wormholes which are geodesically complete by *construction*, as a result of a copy-and-paste procedure of two well behaved spacetimes, even though there are divergent physical quantities precisely at the thin-shell where the throat is located. In this sense, it is indispensable to perform a detailed analysis of the structure of these spacetimes. In previous works by some of us, it was shown that these kinds of wormhole spacetimes are actually geodesically complete in all cases [53, 54] and that the potential existence of arbitrarily large tidal forces across the wormhole does not imply the loss of causal contact among the constituents of travelers going through the wormhole throat [55].

The problem of scalar waves propagating into a wormhole was also analyzed [53] confirming the well defined behavior of these spacetimes and therefore reinforcing the view that physical observers traveling through these geometries do not experience any destructive effects. Then, in all respects, the wormhole spacetimes presented can be regarded as non-singular. In this context, it was also shown that null geodesics reach the wormhole in a finite

time of the affine parameter in the case of Born-Infeld gravity but in an infinite affine time in the $f(R)$ gravity since the latter wormholes lie in the future (or past) boundary of the spacetime [35]. On the other hand, depending on the mass-to-charge ratio, these solutions can be free of horizons which in turn ensures that the wormholes are really traversable. Remarkably, these features have been obtained not as a consequence of any designer or engineering process but as the result of a direct derivation from well motivated gravitational actions. The key-stone seems to be the Palatini framework where the original theories were formulated.

In summary, the wormhole spacetimes presented in this work arise in a more natural (less artificial) way in Palatini formalism where metric and connection are assumed as independent dynamical variables. This novel approach may bring up new avenues to enlarge our knowledge of wormhole physics, offer new insights for constructing wormhole solutions in the context of GR, and find reasonable scenarios where they might take place.

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